***TRIGONOMETRY***

***Summary:***

***1.*** *Trigonometry deals with the relationships between the sides and angles of a triangle*

***2.*** *A right angled triangle has the following sides relative to angle* ***θ:***

***θ***

***Opposite***

***Adjacentente***

***Hypotenuse***

***3.*** *For any angle* ***θ*** *in a right angled triangle****:***

***(i) (ii) (iii)***



***4.*** *To remember the above ratios****,*** *use the relation* ***SOH−CAH−TOA***

***EXAMPLES:***

***1.*** *Study the triangle below****:***

***θ***

***4cm***

***5cm***

***β***

***3cm***

*Write down the ratios for****: (i) (ii) (iii)*** ***(iv) (v) (vi)***



***2.*** *Using a calculator****,*** *find the value of****:***

***(i) Sin 30° (ii) Cos 45° (iii) Tan 60°***  ***(v) Sin 50° (vi) Cos 75° (vii) Tan 45°***

***3.*** *Using a calculator****,*** *find the value of* ***θ*** *if****:***

***(i) (ii) (iii)*** ***(iv) (v) (vi)***



***4.*** *Find the size of the angle marked* ***θ*** *in the following triangles****:***

***θ***

***8⋅4cm***

***3⋅6cm***

***θ***

***4⋅5cm***

***10cm***

***(i) (ii)***

***(iii) (iv)***

***θ***

***3cm***

***5cm***

***θ***

***7cm***

***14cm***

***5.*** *Find the length of the side marked* ***x*** *in the following triangles****:***

***(i) (ii)***

***8cm***

***x***

***48°***

***x***

***14cm***

***36°***

***(iii) (iv)***

***6cm***

***x***

***30°***

***x***

***12cm***

***60°***

***6.*** *Find the length marked* ***x*** *in the following figures****:***

***10cm***

***32°***

***38°***

***x***

***P***

***Q***

***R***

***S***

***R***

***Q***

***P***

***52°***

***25°***

***12cm***

***S***

***x***

***(i) (ii)***

***(iii) (iv)***

***50°***

***30°***

***P***

***Q***

***R***

***S***

***12cm***

***x***

***x***

***R***

***Q***

***P***

***40°***

***10°***

***S***

***25cm***

***S***

***R***

***Q***

***P***

***14cm***

***x***

***35°***

***10°***

***60°***

***25°***

***P***

***Q***

***R***

***S***

***9cm***

***x***

***(v) (vi)***

***47cm***

***S***

***R***

***Q***

***P***

***x***

***30°***

***35°***

***(vii)***

***7.*** *A ladder* ***12m*** *long leans against a wall and makes an angle of* ***30°*** *with the wall****.*** *Find****:***

***(i)*** *how high up the wall does the ladder reach*

***(ii)*** *how far from the wall is the foot of the ladder*

***8.*** *Find the area of a rectangle whose diagonal is* ***10cm*** *long and makes an angle of* ***55°*** *with one of the sides*

***9.*** *An isosceles triangle has a base of* ***16cm*** *and a vertical angle of* ***64°.*** *Find the height and area of the triangle*

***10.*** *Find the area of a regular pentagon of side* ***10cm***

***Hint:*** *Divide the figure into* ***5*** *triangles and find the angle of each at the centre Required area = 5 × area of one triangle*

***12.*** *Find the length of the shadow of a vertical pole* ***60cm*** *tall when the sun is at* ***56°*** *to the horizontal*

***13.*** *A ship sails* ***35km*** *on a bearing of* ***042°.*** *Calculate****: (i)*** *how far east has it travelled* ***(ii)*** *how far north has it travelled*

***14.*** *A ship sails* ***35km*** *on a bearing of* ***243°.*** *Calculate****: (i)*** *how far south has it travelled* ***(ii)*** *how far west has it travelled*

***15.*** *A ladder* ***9m*** *long leans against a wall with its foot* ***5m*** *away from the wall****.*** *Find the angle between the ladder and the wall*

***16.*** *An isosceles triangle has sides of length* ***8cm, 8cm*** *and* ***5cm.*** *Find the angle between the two equal sides*

***17.*** *Find the acute angle between the diagonals of a rectangle whose sides are* ***5cm*** *and* ***8cm***

***18.*** *A man walks from town* ***P 9 km*** *due north then* ***12km*** *due east to town* ***Q.*** *Calculate the distance and bearing of* ***P*** *from* ***Q***

***19.*** *A boat sails* ***15km*** *on a bearing of* ***000°.*** *It then sails* ***8km*** *due East****.*** *Calculate the distance and bearing of the ship from its starting point*

***20.*** *Two ships set off from port* ***P*** *at the same time****.*** *One ship sails* ***8km*** *on a*

*bearing of* ***030°*** *to reach point* ***Q*** *and the other ship sails* ***15km*** *on a bearing*

*of* ***120°*** *to reach point* ***R.*** *Calculate the****:***

***(i)*** *distance and bearing of* ***R*** *from* ***Q***

***(ii)*** *area of the figure bounded by* ***P QR***

***EER:***

***1.*** *A road is inclined at* ***30°*** *to the horizontal****.*** *Find the vertical height climbed when a car travels* ***800m*** *up the road*

***2.*** *The stretched string of a kite is* ***120m*** *long and makes an angle of* ***30°*** *with the horizontal****.*** *Find the height of the kite above the ground*

***3.*** *Find the volume of a cone whose vertical angle is* ***60°*** *and slant side* ***14cm*** *long*

***4.*** *Find the area of a regular nonagon of side* ***10cm***

***5.*** *In the figure below****, AD*** *is perpendicular to* ***BC, AD = DB, AC = 9cm*** *and angle* ***CAD = 30°***

***30°***

***A***

***B***

***C***

***DS***

***9cm***

*Find the length of* ***AB***

***6.*** *The figure below shows a prism* ***ABCDEF*** *of uniform triangular cross−section* ***ABF*** *in which* ***FG*** *is perpendicular to* ***AB*** *such that* ***∠AFG = 30°, ∠AFB = 80°***

***50°***

***30°***

***12cm***

***20cm***

***A***

***B***

***C***

***D***

***F***

***E***

***G***

*Given that* ***AF = 12cm*** *and* ***BC = 20cm,*** *find the volume of the prism*

***6. ABCD*** *is a quadrilateral in which* ***AB = 6cm, AC = 10cm, CD = 5cm,*** *angles* ***ABC*** *and* ***CDA*** *are* ***90°*** *each****.*** *Calculate the****:***

***(i)*** *length of* ***BC***

***(ii)*** *size of angle* ***ACD***

***7.*** *In the figure below****,*** *the angles* ***VAC, VAB*** *and* ***BAC*** *are all* ***90°.*** *Find the value of* ***h,*** *if* ***BC = 60cm***

***60cm***

***C***

***B***

***30°***

***h***

***A***

***38°***

***V***

***8.*** *A ladder* ***13m*** *long leans against a wall with its foot* ***5m*** *away from the wall****.*** *Find****:***

***(i)*** *how high up the wall does the ladder reach*

*(ii) the angle between the ladder and the wall*

***9.*** *If* ***SQ = 9cm,*** *find the length of* ***PR*** *in the diagram below*

***S***

***R***

***Q***

***P***

***40°***

***30°***

***9cm***

***10.*** *If* ***CD = 8cm,*** *find the length of* ***AB*** *in the diagram below*

***8cm***

***B***

***D***

***C***

***A***

***45°***

***60°***

***11.*** *If* ***AB = 10cm,*** *find the length of* ***AD*** *and* ***BD*** *in the diagram below*

***10cm***

***A***

***D***

***C***

***B***

***30°***

***50°***

***12.*** *If* ***AB = 14cm,*** *find the length of* ***BC*** *in the diagram below*

***14cm***

***B***

***C***

***D***

***A***

***15°***

***50°***

***13.*** *If* ***BD = 40cm,*** *find the length of* ***AB*** *and* ***CD*** *in the diagram below*

***D***

***C***

***B***

***A***

***30°***

***38°***

***40cm***

***14.*** *A rectangular piece of cardboard measuring* ***27cm*** *long and* ***15cm*** *wide rests against a vertical wall as shown below*

***25°***

***27cm***

***15cm***

***A***

***B***

***C***

***D***

*Given that angle* ***DAY = 25°,*** *find the height of* ***C*** *above the ground*

***15.*** *If* ***QR = 15cm,*** *find the length of* ***PS*** *and* ***PQ*** *in the diagram below*

***40°***

***20°***

***P***

***Q***

***R***

***S***

***15cm***

***16.*** *A regular hexagon of side* ***8cm*** *form the cross section of a prism* ***20cm*** *long****.***  *Find the****:***

***(i)*** *area of the cross section of the prism*

***(ii)*** *volume of the prism*

***17.*** *In the figure below****,*** *chord* ***AB*** *subtends an angle of* ***120°*** *at the centre* ***O*** *of the circle whose radius is* ***7cm***

***7cm***

***120°***

***A***

***B***

***O***

*Find the****:***

***(i)*** *shortest distance of chord* ***AB*** *from the centre*

***(ii)*** *perimeter of the shaded segment*

***(iii)*** *perimeter of the region enclosed between chord* ***AB*** *and the major arc*

***ANGLES OF ELEVATION AND DEPRESSION***

***Summary:***

***(i)*** *Angle of elevation is the angle above the horizontal to see an object upwards*

***(ii)*** *Angle of depression is the angle below the horizontal to see an object downwards*

***(iii)*** *The above angles are illustrated as follows****:***

***Angle of elevation***

***Angle of depression***

***EXAMPLES:***

***1.*** *The angle of elevation of the top of the tower from a point* ***12m*** *away from its foot is* ***60º.*** *Calculate the height of the tower*

***2.*** *The angle of depression of a boat from the top of a tower* ***25m*** *high is* ***30º.*** *Find how far is the boat from the foot of the tower*

***3.*** *A man* ***1⋅5 m*** *tall is* ***20 m*** *away from a tower* ***30 m*** *high****.*** *Find the angle of elevation of the top of the tower*

***4*** *When a man* ***1⋅8 m*** *tall is* ***12 m*** *away from a tower****,*** *the angle of elevation of the top of the tower is* ***30º.*** *Find the height of the tower****.***

***5.*** *From a point* ***40m*** *away from the foot of the building****,*** *the angles of elevation of the top and bottom of a flagpole on top of a building are* ***58º*** *and* ***50º*** *respectively****.*** *Find the****:******(i)*** *height of the building* ***(ii)*** *height of the flagpole*

***6.*** *The angles of elevation of the top of the tower from points* ***P*** *and* ***Q******15m*** *apart on the same side of the tower are* ***60º*** *and* ***40º*** *respectively****.*** *Calculate the****: (i)*** *height of the tower* ***(ii)*** *distance of* ***P*** *from the foot of the tower*

***7.*** *The angles of elevation of the top of the tower from points* ***P*** *and* ***Q******54m*** *apart on either side of the tower are* ***45º*** *and* ***30º*** *respectively****.*** *Calculate the****: (i)*** *height of the tower* ***(ii)*** *distance of* ***P*** *from the foot of the tower*

***8.*** *Building* ***A 12⋅5 m*** *high is* ***43 m*** *away from building* ***B.*** *When a man* ***1⋅5 m*** *tall stands on top of* ***A*** *and* ***B*** *respectively****,*** *the angles of depression of a point* ***P*** *between the buildings are* ***30º*** *and* ***40º.*** *Find the height of building* ***B.***

***9.*** *From the top of a building* ***100m*** *high the angles of depression of the top and bottom of a tower are* ***30º*** *and* ***60º*** *respectively****.*** *Find the height of the tower*

***10.*** *From the top of a tower* ***50m*** *high the angles of elevation and depression of the top and bottom of a building are* ***20º*** *and* ***27º*** *respectively****.*** *Find the height of the building*

***11.*** *From the bottom and top of a tower* ***60m*** *high the angles of elevation of the top of a building are* ***60º*** *and* ***30º*** *respectively****.*** *Find the height of the building*

***12.*** *A flag mast slants towards the west at an angle of* ***13º*** *the vertical. From a point* ***Q*** *to the east and* ***20 m*** *away from the foot,* ***F*** *of the mast****,*** *the angle of elevation of the top* ***T*** *of the mast is* ***35º****. From another point* ***R*** *to the west of the mast, the angle of elevation of the top* ***T*** *is* ***22º****.If* ***Q, F*** *and* ***R*** *are on level ground, Find the****:***

***(i)*** *vertical distance of the top* ***T*** *from the ground*

***(ii)*** *distance of the foot of the mast* ***F*** *from* ***R***

***(iii)*** *length* ***TF***

***EER:***

***1.*** *The angle of depression of a boat from the top of a tower* ***40m*** *high is* ***30º.*** *Find how far is the boat from the foot of the tower*

***2.*** *Find the length of the shadow of a vertical pole* ***60m*** *tall when the angle of elevation of the sun is* ***56° .***

***3.*** *The angle of elevation of the top of a tower to a man* ***1⋅7 m*** *tall and* ***20 m*** *away from the tower is* ***43º.*** *Find the height of the tower****.***

***4.*** *The angles of elevation of the top of the tower are* ***30º*** *and* ***50º*** *from two points* ***10m*** *apart on the same side of the tower****.*** *Find how tall is the tower*

***5.*** *The shadow of a vertical post increases by* ***10m*** *when the angle of elevation of the sun changes from* ***45°*** *to* ***30° .*** *Find the height of the post****.***

***6.*** *The angles of elevation of the top of a tower* ***80m*** *high to two men standing on either sides of the tower are* ***45º*** *and* ***60º*** *respectively****.*** *Find the distance between the two men*

***7.*** *Find the height of a vertical post that casts a shadow* ***20m*** *long when the angle of elevation of the sun is* ***53° .***

***8.*** *The angle of elevation of the top of a tower* ***50m*** *high from the foot of a hill is* ***30º*** *and angle of elevation of the top of a hill from the foot of a tower is* ***60º.*** *Calculate the height of the hill*

***9.*** *From the top of a tower* ***90m*** *high****,*** *the angles of depression of two ships on either sides of the tower are* ***30º*** *and* ***45º.*** *Find the distance between the two ships*

***10.*** *From the top of a house* ***40m*** *high****,*** *the angles of depression of the top and bottom of a tower are* ***30º*** *and* ***60º.*** *Find the height of the tower*

***11.*** *From the top of a house* ***80m*** *high****,*** *the angles of elevation and depression of the top and bottom of a hill are* ***60º*** *and* ***30º*** *respectively****.*** *Find the height of the hill*

***12.*** *On a shore running from east to west are two ports* ***P*** *and* ***Q*** *which are* ***18km*** *apart****.*** *Town* ***R*** *on an island on the same level as* ***P*** *and* ***Q*** *is on a bearing of* ***230°*** *from* ***P*** *and* ***140°*** *from* ***Q*** *respectively****.*** *A pilot flying a plane above port* ***P*** *observes town* ***R*** *at an angle of depression of* ***6°.*** *Calculate the****: (i)*** *distances* ***PR*** *and* ***QR (ii)*** *vertical height of the plane above* ***P (iii)*** *angle of elevation of the plane from port* ***Q***

***13.*** *Three towns* ***A, B*** *and* ***C*** *lie on the same level ground****.*** *Town* ***B*** *is* ***15km*** *away from town* ***C.*** *The bearings of towns* ***B*** *and* ***C*** *from* ***A*** *are* ***060°*** *and* ***150°*** *respectively****.*** *The bearing of* ***C*** *from* ***B*** *is* ***200°.*** *To a pilot flying an aircraft above* ***A,*** *the angle of depression of* ***C*** *is* ***7⋅5°.*** *Calculate the****: (i)*** *distances* ***AB*** *and* ***AC (ii)*** *vertical height of the aircraft above* ***A (iii)*** *angle of elevation of the aircraft from* ***B***

***14.*** *From the top of a tower* ***50m*** *high****,*** *the angles of depression of two boats are* ***45º*** *and* ***30º*** *respectively****.*** *Find the distance between the boats****,*** *if they are****: (i)*** *on the same side of the tower* ***(ii)*** *on either sides of the tower*

***15.*** *The angle of elevation of the top of a tower from a point* ***P*** *due south of the tower is* ***38º*** *and from another point* ***Q*** *due east of the tower is* ***29º.*** *Find the height of the tower****,*** *if distance* ***PQ = 50m.***

***TRIGONOMETRY OF SPECIAL ANGLES***

***Summary:***

***(i)*** *The exact trigonometric ratios of* ***30° 45°*** *and* ***60°*** *can be obtained without a calculator*

***(ii)*** *An equilateral triangle split into two right angled triangles can be used to work out the sine****,*** *cosine and tangent of* ***30°*** *and* ***60°***

***(iii)*** *A right angled isosceles triangle can be used to work out the sine, cosine and tangent of* ***45°***

***EXAMPLES:***

***1.*** *Without using tables or a calculator****,*** *find the value of****:******(i) Sin 30° (ii) Cos 30° (iii) Tan 30°***  ***(v) Sin 60° (vi) Cos 60° (vii) Tan 60° Soln Hint:*** *Use an equilateral triangle with sides* ***2*** *units long*

***2.*** *Without using tables or a calculator****,*** *find the value of****:******(i) Sin 45° (ii) Cos 45° (iii) Tan 45°***

***Soln Hint:*** *Use a right angled isosceles triangle with two sides of unit length*

***3.*** *Without using tables or a calculator****,*** *find the value of****:******(i) (ii) 2Sin 30° + 3Cos 60° − Tan 45°***



***(iii)***



***4.*** *Use the fact that*  *to express as a surd and simplify*



***EER:***

***1.*** *With the help of an equilateral triangle and a right angled isosceles triangle****,*** *copy and complete the table below****:***

|  |  |  |  |
| --- | --- | --- | --- |
| ***θ*** | ***30°*** | ***45°*** | ***60°*** |
| ***Sinθ*** |  |  |  |
| ***Cosθ*** |  |  |  |
| ***Tanθ*** |  |  |  |

***2.*** *Express as a surd and simplify****:*** 

***3.****With the help of an equilateral triangle****,*** *show that . Hence without using tables or calculators****,*** *find the value of* 

***THE SINE AND COSINE RULE***

***Summary: 1.*** *The sine and cosine rules are used to solve problems involving any triangle* ***2.*** *A general triangle has the following sides relative to its angles****:***

***A***

***B***

***C***

***a***

***b***

***c***

***3. (i)****The sine rule relates the sides and angles of any triangle as follows****:***



***(ii)****The sine rule is used when we are given an angle and its opposite side*

***4. (i)****The cosine rule relates the sides of any triangle and one of its angles as follows****:***



***(ii)****The cosine rule is used when we are given two sides and the included angle or three sides*

***5.*** *The area of any triangle Or Or*



***6.*** *In any triangle****: (i)*** *The three angles add up to* ***180° (ii)*** *The largest angle is always opposite the longest side* ***(ii)*** *The smallest angle is always opposite the shortest side*

***7.*** *The radius of a circle circumscribing any triangle is obtained using the relation****:***



***EXAMPLES:***

***1.*** *Find the size of angle* ***PRQ*** *in the following diagrams****:***

***R***

***Q***

***P***

***4cm***

***4⋅2cm***

***6⋅9cm***

***R***

***9cm***

***4cm***

***75°***

***P***

***Q***

***(i) (ii)***

***2.*** *Find the length of side* ***YZ*** *in the following diagrams****:***

***Y***

***3cm***

***35°***

***X***

***Z***

***7cm***

***Y***

***4cm***

***40°***

***Z***

***X***

***95°***

***(i) (ii)***

*The angles in a triangle add to* ***180°***

***⇒ X = ∠ 45°***

***3.*** *Find the area of the given triangle below****:***

***12cm***

***8cm***

***30°***

***4.*** *A triangle has sides of length* ***3cm, 5cm*** *and* ***7cm.*** *Find the****: (i)*** *size of its largest angle*  ***(ii)*** *area of the triangle* ***(iii)*** *radius of the circle circumscribing the triangle.*

***5.*** *Find the length of* ***CD*** *in the given figure below*

***A***

***50°***

***35°***

***60°***

***B***

***C***

***D***

***12m***

***6.*** *The figure* ***PQRS*** *below represents a quadrilateral piece of land divided into three triangular plots such that* ***QT = 100m, RS = 80m,*** *angle* ***PQT = 30º,*** *angle* ***PRT = 45º*** *and angle* ***PRS = 100º***

***100m***

***P***

***R***

***T***

***Q***

***30°***

***45°***

***80m***

***100°***

***S***

*Calculate the****: (i)*** *length of* ***PT*** *and* ***PS*** *correct to* ***4*** *significant figures*  ***(ii)*** *perimeter of the land* ***(iii)*** *area of the land*

***7.*** *The points* ***P,Q*** *and* ***R*** *are on level ground****.*** *A vertical flagpole* ***ST*** *stands between* ***P*** *and* ***Q*** *such that* ***Q*** *is* ***15 m*** *away from* ***S****, the base of the pole****.*** *The angles of elevation of* ***T*** *from* ***P*** *and* ***Q*** *are* ***48º*** *and* ***36º*** *respectively. If angle* ***PQR = 35º*** *and* ***QR = 13m,*** *calculate the****:***

***(i)*** *height of the flagpole* ***ST,***

***(ii)*** *length* ***PQ,***

***(iii)*** *angle of elevation of* ***T*** *from* ***R.***

***8.*** *Two ships set off from port* ***P*** *at the same time****.*** *One ship sails* ***70km*** *on a*

*bearing of* ***050°*** *to reach point* ***Q*** *and the other ship sails* ***150km*** *on a bearing*

*of* ***110°*** *to reach point* ***R.***

***(a)*** *Calculate the****:***

***(i)*** *distance and bearing of* ***R*** *from* ***Q***

***(ii)*** *area of the figure bounded by* ***P QR***

***(b)*** *If both ships take* ***t*** *hours to reach their destination and the speed of the*

*faster ship is find the****:***



***(i)*** *value of* ***t***

***(ii)*** *speed of the slower ship*

***9.*** *Port* ***B*** *is* ***25 km*** *east of port* ***C****. A navigator observes that the bearing of* ***C*** *from*

*his ship is* ***310º*** *and that of* ***B*** *is* ***018º.***

***(a)*** *Calculate the****:***

***(i)*** *distance and bearing of the ship from* ***B***

***(ii)*** *distance and bearing of the ship from* ***C***

***(b)*** *If the ship begins to sail at a speed of* ***10 kmh-1*** *on the bearing of* ***240º****,*

*determine the distance and bearing of the ship from* ***C*** *after* ***48 minutes.***

***10.*** *In the figure below****,*** *chord* ***AB*** *subtends an angle of* ***120°*** *at the centre* ***O*** *of the circle whose radius is* ***7cm***

***7cm***

***120°***

***A***

***B***

***O***

*Find the****:***

***(i)*** *length of**chord* ***AB***

***(ii)*** *area of the shaded segment*

***11.*** *The figure below shows a water trough cut from a horizontal cylindrical tank of length* ***250cm*** *and radius* ***70cm.******AB*** *and* ***CD*** *are chords* ***60cm*** *long and below the centre of the tank*

***B***

***250cm***

***60cm***

***A***

***C***

***D***

*Find the****:***

***(i)*** *area of the cross section of the trough*

***(ii)*** *volume of water in litres required to fill the trough*

***EER:***

***1.*** *Calculate the area of triangle* ***ABC*** *in which* ***AB = 5cm, AC = 4cm*** *and angle* ***BAC = 150°.***

***2.*** *In the figure below****,******PQ = 8cm,******QR = 6cm,******RS = 12cm,*** *angle* ***PQR = 90°*** *and angle* ***PRS = 130°.***

***6cm***

***8cm***

***12cm***

***130°***

***P***

***S***

***Q***

***R***

*Calculate the area of the quadrilateral* ***PQRS***

***3.*** *In triangle* ***ABC,*** *angle* ***BAC = 120°, BC = 20cm*** *and* ***AC = 8cm.*** *Find the size of angle* ***ABC***

***4.*** *A boat sails* ***7km*** *on a bearing of* ***306°*** *and then a further* ***11km*** *on a bearing of* ***070°.*** *Calculate the distance and bearing of the return journey*

***5.*** *A triangle has sides of length* ***4cm, 4⋅2cm*** *and* ***6⋅9cm.*** *Find the****: (i)*** *size of its largest angle*  ***(ii)*** *area of the triangle*

***6.*** *Calculate the area of a triangle with sides of length* ***5cm, 7cm*** *and* ***9cm.***

***7.*** *A boat sails* ***22km*** *on a bearing of* ***042°*** *and then a further* ***30km*** *on a bearing of* ***090°.*** *Calculate the distance and bearing of the return journey*

***8.*** *Calculate the area of a triangle whose sides are* ***5cm, 7cm*** *and the angle between them is* ***135°.***

***9. PQR*** *represents a triangular plot of land in which* ***PQ = 36m,******PR = 44m*** *and*

*angle* ***QPR = 68°.*** *Calculate the*

***(i)*** *length of* ***QR***

***(ii)*** *size of angle* ***PRQ***

***(iii)*** *area of the plot*

***10.*** *From port* ***P,*** *ship* ***Q*** *lies* ***11km*** *away on a bearing of* ***041°*** *and* ***R*** *lies* ***8km*** *away*

*on a bearing of* ***341°.*** *Calculate the****:***

***(i)*** *distance and bearing of* ***Q*** *from* ***R***

***(ii)*** *area of the figure bounded by* ***P QR***

***11. ABCD*** *is a quadrilateral in which* ***AB = 4cm, BC = 5cm, CD = 10cm,*** *angle*

***ABC = 80°*** *and angle* ***ACD = 30°.*** *Calculate the****:***

***(i)*** *length of* ***AC***

***(ii)*** *size of angle* ***ACB***

***(iii)*** *length of* ***AB***

***(iv)*** *area of quadrilateral* ***ABCD***

***12.****In the diagram below****, ABCD*** *is a rectangle with* ***CF = 10cm, EF = 8cm,*** *angle* ***BFC = 50°*** *and angle* ***EFA = 52°.***

***E***

***A***

***50°***

***52°***

***10cm***

***8cm***

***B***

***F***

***C***

***D***

*Calculate the****:***

***(i)*** *length of* ***BC*** *and* ***AB***

***(ii)*** *area of triangle* ***CEF***

***13.*** *A point* ***P*** *is* ***10km*** *due north of* ***Q.*** *A man walks from* ***Q*** *on a bearing of* ***030°.*** *Calculate how far he travels before he is****: (i)*** *equidistant from* ***P*** *and* ***Q (ii)*** *as close as possible to* ***P (iii)*** *north east of* ***P***

***14. ABCD*** *is a quadrilateral in which* ***AB = 7cm, BC = 6cm, DA = 4cm,*** *angle*

***BAD = 60°*** *and angle* ***BCD = 90°.*** *Calculate the****:***

***(i)*** *length of* ***BD*** *and* ***CD***

***(ii)*** *size of angle* ***ADC***

***(iii)*** *length of* ***AB***

***(iv)*** *area of quadrilateral* ***ABCD***

***15.*** *In a triangle* ***ABC,******AC = 8 cm,******BC = 7cm*** *and* ***AB = 12cm.*** *Find the****:***

***(i)*** *largest angle of the triangle,*

***(ii)*** *size of angle* ***QPR***

***(iii)*** *area of triangle.*

***(iv)*** *radius of the circumcircle of the triangle.*

***16.*** *An equilateral triangle is inscribed in a circle of radius* ***6 cm.*** *Calculate the length of the side of the triangle****.***

***SOLVING TRIGONOMETRIC EQUATIONS***

***Summary:***

***1.*** *Trigonometric equations are solved using both a calculator and a quadrant diagram* ***2.*** *A quadrant diagram is a circle centred at the origin in the x−y plane and marked as follows****:***

***180°***

***270°***

***360°***

***90°***

***0°***

***+ve x***

***−ve x***

***+ve y***

***−ve y***

***A***

***C***

***T***

***S***

***A =*** *All the ratios are positive****, C =*** *Only cosine is positive****, T =*** *Only tangent is positive****,******S =*** *Only sine is positive*

***3.*** *The term* ***ACTS*** *is used to remember the quadrants*

***4.*** *Angles are measured from the positive x−axis in an anticlockwise direction*

***5.*** *The acute angle subtended with the x−axis in any quadrant is called a reference angle*

***6.*** *Angles greater than* ***90°*** *and their corresponding reference angles have the same trigonometric function values* ***(****The signs may differ depending on the quadrant enclosing the reference angle****)***

***7.*** *In the quadrant diagram below****,******45°*** *is the reference angle of* ***135°***

***45°***

***135°***

***A***

***C***

***T***

***S***

*From* ***(6)*** *above it follows that****:***

***(i)***



***(ii)***



***(iii)***



***EXAMPLES:***

***1.*** *Without using tables or a calculator****,*** *find the value of****:******(i) Sin 150° (ii) Cos 150° (iii) Tan 150°***  ***(v) Sin 315° (vi) Cos 315° (vii) Tan 315° (vii) Sin 240° (viii) Cos 240° (ix) Tan 240°***  ***(x) Sin 390° (xi) Cos 390° (xii) Sin 510° (xiii) Cos 510° (xiv) Tan 510°***  ***(xv) Cos780° (xvi) tan765° Soln Hint:*** *Express the given angle in terms of its reference angle on a quadrant diagram*

***2.*** *Given that* ***sinθ = 0⋅5*** *for* ***0° < θ < 360°,*** *find the two possible values of* ***θ***

***Soln Hint:*** *The first angle is got from a calculator and the rest from a quadrant diagram*

***3.*** *Solve the equation* ***2cosθ − 1 = 0*** *for* ***0° < θ < 360°***

***4.*** *Solve the equation for* ***0° < θ < 360°***



***5.*** *Solve the equation for* ***0° < θ < 360°***



***6.*** *Given that* ***sinθ = −0⋅5*** *for* ***0° < θ < 360°,*** *find the two possible values of* ***θ***

***Soln Hint:*** *First ignore the negative sign and find the reference angle from a calculator*

*Use this angle in the quadrants where the sine function is negative*

***7.*** *Solve the equation* ***2cosθ + 1 = 0*** *for* ***0° < θ < 360°***

***8.*** *Solve the equation* ***tanθ = − 1*** *for* ***0° < θ < 360°***

***9.*** *Given that for* ***0° < θ < 360°,*** *find the four possible values of* ***θ***



***10.*** *Given that*  *and* ***θ*** *is obtuse****,*** *find without using tables or a calculator the value of****:***



***(i) cosθ***

***(ii)******tanθ***

***Soln Hint:*** *The sine function is positive in both quadrants* ***A*** *and* ***S*** *but since* ***θ*** *is obtuse****,*** *then quadrant* ***A*** *is eliminated*

***11.*** *Given that*  *for* ***0° < θ < 270°,*** *find without using tables or a calculator the value of****:***



***(i) sinθ***

***(ii) cosθ***

***(iii) sinθ*** *−* ***cosθ***

***(iv)***



***Soln Hint:*** *The tangent function is negative in both quadrants* ***S*** *and* ***C*** *but since* ***θ*** *lies between* ***0°***  *and* ***270°,*** *then quadrant* ***C*** *is eliminated*

***12.*** *Given that*  *for* ***90° < θ < 270°,*** *find without using tables or a calculator the value of* ***sinθ*** *−* ***cosθ***



***Soln Hint:*** *The tangent function is positive in both quadrants* ***A*** *and* ***T*** *but since* ***θ*** *lies between* ***90°***  *and* ***270°,*** *then quadrant* ***A*** *is eliminated*

***13.*** *Given that*  *for* ***180° < θ < 360°,*** *find without using tables or a calculator the value of****:***



***(i) sinθ***

***(ii) tanθ***

***(iii) cosθ*** *−* ***sinθ***

***Soln Hint:*** *The cosine function is negative in both quadrants* ***S*** *and* ***T*** *but since* ***θ*** *lies between* ***180°***  *and* ***360°,*** *then quadrant* ***S*** *is eliminated*

***14.*** *Given that*  *and* ***θ*** *is reflex****,*** *find without using tables or a calculator the values of****:***



***(i)***



***(ii)***



***Soln Hint:*** *The tangent function is negative in both quadrants* ***S*** *and* ***C*** *but since* ***θ*** *is reflex****,*** *then quadrant* ***S*** *is eliminated*

***15.*** *Given that find without using tables or a calculator the possible values of* ***cosθ*** *−* ***sinθ***



***Soln Hint:*** *The tangent function is positive in both quadrants* ***A*** *and* ***T.*** *Thus both quadrants are valid since* ***θ*** *has no restriction*

***16.*** *Given that*  *and* *where* ***A*** *and* ***B*** *are both acute angles****,*** *find without using tables or a calculator the value of*



***Soln Hint:*** *Angles* ***A*** *and* ***B*** *are both acute in quadrant* ***A***

***EER:***

***1.*** *Without using tables or a calculator****,*** *find the value of****:******(i) Sin 225° (ii) Cos 225° (iii) Tan 225°***  ***(v) Sin 300° (vi) Cos 300° (vii) Tan 300° (vii) Sin 240° (viii) Cos 750° (ix) Tan 210°***  ***(x) Sin 210° (xi) Cos 480° (xii) Sin 330°***

***2.*** *Solve the equation for* ***0° < θ < 360°***



***3.*** *In triangle* ***ABC,*** *angle* ***BAC = 20°, BC = 3cm*** *and* ***AC = 5cm.*** *Find the two possible values of angle* ***ABC***

***4.*** *If* ***cos x = − 0⋅634*** *for* ***90° < x < 270°,*** *find the two possible values of* ***x***

***5.*** *If* ***3tan x − 2 = 4cos35°*** *for* ***0° < x < 360°,*** *find the two possible values of* ***x***

***6.*** *Given that* ***tan35° = 0⋅7,*** *without using tables or a calculator****,*** *find the value of****:******(i) tan145° (ii) 2tan215° + 10tan325°***

***7.*** *Find the two possible angles in the range* ***0o*** *to* ***360o*** *which satisfy the equations****:******(i)******sinx = 0⋅4210******(ii) cosx = −0⋅8660 (iii) tanx = 2⋅106***

***8.*** *Given that for* ***0° < θ < 360°,*** *find the four possible values of* ***θ***



***9.*** *Given that*  *and* ***θ*** *is acute****,*** *find without using tables or a calculator the values of* ***cosθ*** *and* ***tanθ***



***10.*** *If* ***cos θ*** ***= − 0⋅5*** *for* ***0° < θ******< 360°,*** *find the two possible values of* ***θ***

***11.*** *Given that*  *for* ***180° < θ < 360°,*** *find without using tables or a calculator the values of* ***sinθ*** *and* ***cosθ***



***12.*** *Without**using tables or calculators find the values of the following in surd form*

***(i)******cos780******(ii) sin315*** ***(iii) tan585***

***GRAPHING TRIGONOMETRIC CURVES***

***Summary:***

***1. (i)*** *The sine and cosine curves have hills and valleys in continuous form*

***(ii)*** *The height of such hills is called the amplitude*

***(iii)*** *The distance from the top of a hill to the next is called the period*

***(iv)*** *The maximum and minimum values of the function occur at its turning points*

***(v)*** *By drawing suitable lines****,*** *the drawn graphs can be used to solve related equations*

***2.*** *Consider a general function* ***y = ASinωx*:**

***(i)*** *Amplitude =* ***|A |***

***(ii)*** *Period*



***EXAMPLES:***

***1.*** *Find the amplitude and period of the function* ***y = −3Sin2x***

***2.******(a)*** *Draw a graph of* ***y = Sinx*** *for* ***0° ≤ x ≤ 360°*** *using intervals of* ***30°***

***(****use a scale of* ***1cm : 30°*** *on the* ***x-axis*** *and* ***2cm : 1 unit*** *on the* ***y-axis)***

***(b)*** *Use your graph to solve the equations****:***

***(i) Sinx = 0***

***(ii)*** ***Sinx = 0⋅5***

***(iii)*** ***Sinx = −0⋅5***

***(c)*** *State the amplitude and period of* ***y = Sinx***

***3.******(a)*** *Draw a graph of* ***y = Cosx*** *for* ***0° ≤ x ≤ 360°*** *using intervals of* ***30°***

***(****use a scale of* ***1cm : 30°*** *on the* ***x-axis*** *and* ***2cm : 1 unit*** *on the* ***y-axis)***

***(b)*** *Use your graph to solve the equations****:***

***(i) Cosx = 0***

***(ii)*** ***Cosx = 0⋅5***

***(iii)*** ***Cosx = −0⋅5***

***(c)*** *State the amplitude and period of* ***y = Cosx***

***3.******(a)*** *Draw a graph of* ***y = 3Cosx − 4Sinx*** *for* ***0° ≤ x ≤ 360°*** *using intervals of*

***30°.* *(****use a scale of* ***1cm : 30°*** *on the* ***x-axis*** *and* ***2cm : 1 unit*** *on the* ***y-axis)***

***(b)******(i)*** *State the minimum and maximum values of* ***3Cosx − 4Sinx***

***(ii)*** *State the value of* ***x*** *at which the maximum value of* ***y*** *occur*

***(c)*** *Use your graph to solve the equations****:***

***(i) 3Cosx − 4Sinx = 0***

***(ii)*** ***3Cosx − 4Sinx + 1 = 0***

***(d)*** *State the range of values of* ***x*** *for which* ***3Cosx − 4Sinx < −4***

***4.******(a)*** *Draw on the same axes****,*** *the graphs of* ***y = Sin2x*** *and* ***y = 3Cosx − 2*** *for*

***0° ≤ x ≤ 360°*** *using intervals of* ***30°.* *(****use a scale of* ***1cm : 30°*** *on the* ***x-axis***

*and* ***2cm : 1 unit*** *on the* ***y-axis)***

***(b)*** *Use your graphs to solve the equation* ***3Cosx − 2 = Sin2x***

***(c)*** *State the amplitude of* ***y = 3Cosx − 2***

***5.******(a)*** *Draw on the same axes****,*** *the graphs of* ***y = 4Co2x*** *and* ***y = 2Sin(2x +30°)*** *for*

***0° ≤ x ≤ 180°*** *using intervals of* ***15°.* *(****use a scale of* ***1cm : 15°*** *on the* ***x-axis***

*and* ***2cm : 1 unit*** *on the* ***y-axis)***

***(b)*** *Use your graphs to solve the equation* ***4Cos2x − 2Sin(2x +30°) = 0***

***(c) (i)*** *State the amplitude of* ***y = 4Cos2x***

***(ii)*** *State the period of* ***y = 2Sin(2x +30°)***

***EER:***

***1. (a)*** *Copy and complete the table below for the function* 

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| ***x*** | ***0*** | ***30*** | ***60*** | ***90*** | ***120*** | ***150*** | ***180*** | ***210*** | ***240*** |
| ***Sinx*** |  | ***0⋅5*** |  |  | ***0⋅87*** |  |  | ***−0⋅5*** |  |
| ***−Cosx*** |  | ***−0⋅87*** |  |  | ***0⋅5*** |  |  | ***0⋅87*** |  |
| ***y*** |  | ***−0⋅37*** |  |  | ***1⋅37*** |  |  | ***0⋅37*** |  |

***(b)*** *Draw a graph of*  *for* ***0° ≤ x ≤ 240°*** ***(****use a scale of* ***2cm : 30°*** *on the* ***x-axis*** *and* ***2cm : 0⋅5 units*** *on the* ***y-axis)***

***(c)*** *Use your graph to solve the equations****:***

***(i) Sinx − Cosx = 0***

***(ii)*** S***inx − Cosx = 1⋅2***

***2.******(a)*** *Draw a graph of* *for* ***0° ≤ x ≤ 360°*** *using intervals of*

***30°.* *(****use a scale of* ***1cm : 30°*** *on the* ***x-axis*** *and* ***2cm : 1 unit*** *on the* ***y-axis)***

***(b)*** *Use your graph to solve the equations****:***

***(i) Sinx + Cosx = 0***

***(ii)*** S***inx + Cosx = 1***

***(iii)*** ***2****S****inx = 1 − 2Cosx***

***3.******(a)*** *On the same axes****,*** *draw the graphs of* ***y = 3Cosx + 2Sinx*** *and* *for* ***0° ≤ x ≤ 90°*** *using intervals of* ***15°.* *(****use a scale of* ***2cm : 15°*** *on the* ***x-axis*** *and* ***2cm : 0⋅5 units*** *on the* ***y-axis)***

***(b)*** *State the minimum and maximum values of* ***3Cosx + 2Sinx***

***(c)*** *Use your graphs to solve the equations****:***

***(i) 3Cosx + 2Sinx = 2⋅5***

***(ii)*** 

***4.******(a)*** *Draw on the same axes****,*** *the graphs of* ***y = 2Cosx*** *and* ***y = Sin(x +30°)*** *for*

***0° ≤ x ≤ 360°*** *using intervals of* ***30°.* *(****use a scale of* ***1cm : 30°*** *on the* ***x-axis***

*and* ***1cm : 0⋅5 units*** *on the* ***y-axis)***

***(b)*** *Use your graphs to solve the equation* ***Sin(x +30°) − 2Cosx = 0***

***(c)*** *State the amplitudes of the functions****:***

***(i)******y = Sin(x +30°)***

***(ii) y = 2Cosx***